

Higher GCSE Mathematics Revision Pack

SHAPE AND SPACE – NON-CALC

Q1.

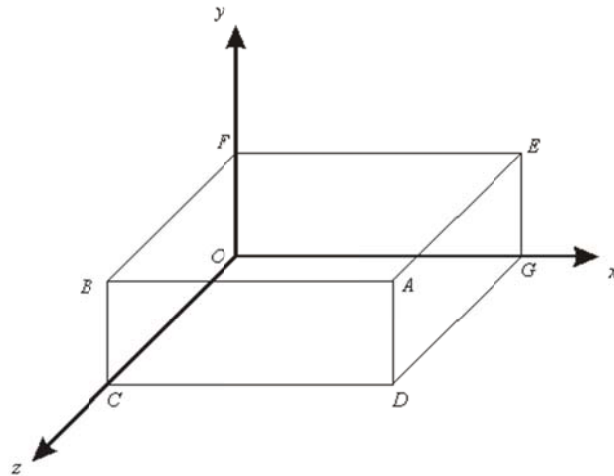


Diagram **NOT** accurately drawn

The diagram shows a cuboid drawn on a 3-D grid.

Vertex A has coordinates $(5, 2, 3)$.

- (a) Write down the coordinates of vertex E .

(..... , ,)

(1)

B and D are vertices of the cuboid.

- (b) Work out the coordinates of the midpoint of BD .

(..... , ,)

(3)

(Total 4 marks)

Q2.

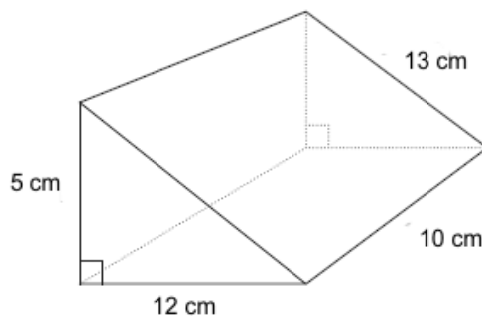


Diagram **NOT** accurately drawn

Work out the total surface area of this triangular prism.

.....

(Total 4 marks)

Q3. The interior angle of a regular polygon is 160° .



Diagram **NOT** accurately drawn

- (i) Write down the size of an exterior angle of the polygon.

.....^o
- (ii) Work out the number of sides of the polygon.

.....

(Total 3 marks)

Q4. The diagram shows 3 sides of a regular polygon.

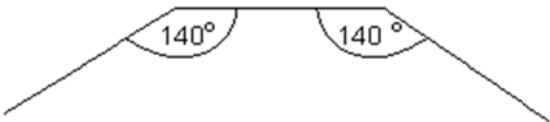


Diagram **NOT** accurately drawn

Each interior angle of the regular polygon is 140° .
 Work out the number of sides of the regular polygon.

.....
(Total 3 marks)

Q5. A piece of card is in the shape of a trapezium.

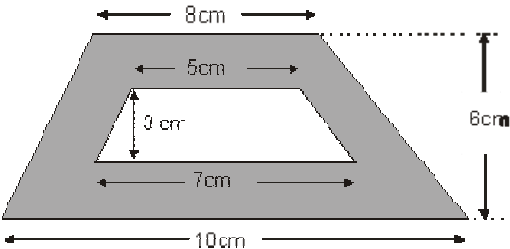


Diagram **NOT** accurately drawn

A hole is cut in the card. The hole is in the shape of a trapezium.
 Work out the area of the shaded region.

..... cm²
(Total 3 marks)

Q6. A plane takes 30 seconds to fly a distance of 8 kilometres.

Work out the average speed of the plane, in miles per hour.

..... miles per hour

(Total 3 marks)

Q7.

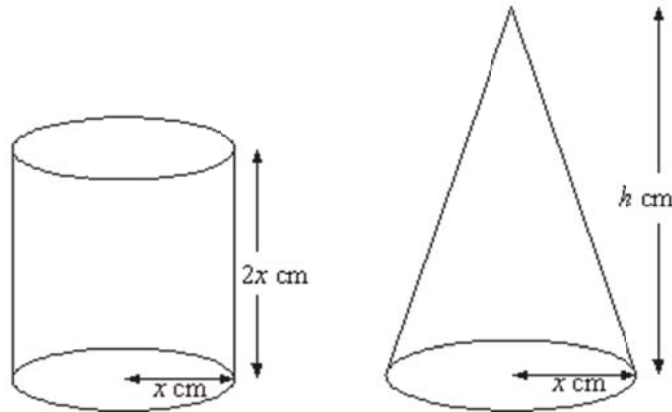


Diagram **NOT** accurately drawn

A cylinder has base radius x cm and height $2x$ cm.

A cone has base radius x cm and height h cm.

The volume of the cylinder and the volume of the cone are equal.

Find h in terms of x .

Give your answer in its simplest form.

$h =$

(Total 3 marks)

Q8.

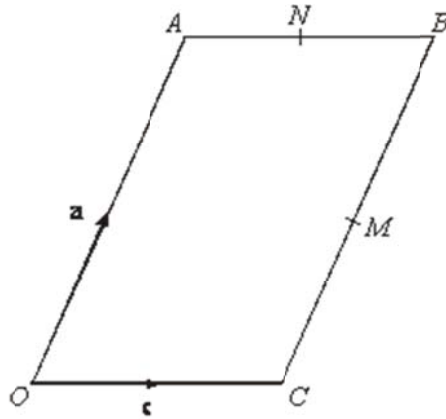


Diagram **NOT** accurately drawn

$OABC$ is a parallelogram.

M is the midpoint of CB .

N is the midpoint of AB .

$$\overrightarrow{OA} = \mathbf{a}$$

$$\overrightarrow{OC} = \mathbf{c}$$

(a) Find, in terms of \mathbf{a} and/or \mathbf{c} , the vectors

(i) \overrightarrow{MB}

.....

(ii) \overrightarrow{MN}

.....

(2)

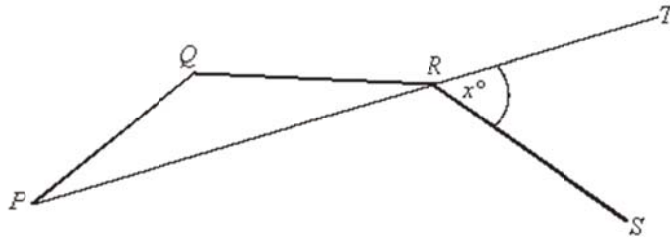
(b) Show that CA is parallel to MN .

(2)

(Total 4 marks)

Q9.

Diagram
NOT
accurately
drawn



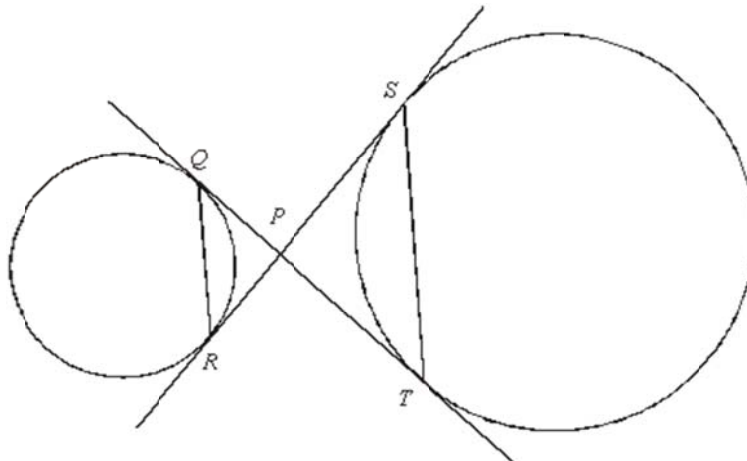
PQ , QR and RS are 3 sides of a regular decagon.
 PRT is a straight line.
 Angle $TRS = x^\circ$

Work out the value of x

$x = \dots\dots\dots$

(Total 5 marks)

Q10.



Q and R are two points on the circumference of a circle.
 S and T are two points on the circumference of another circle.

QT and SR are tangents to both circles. P is the point of intersection of the two tangents.

Prove that QR is parallel to ST .

(Total 5 marks)

Q11.

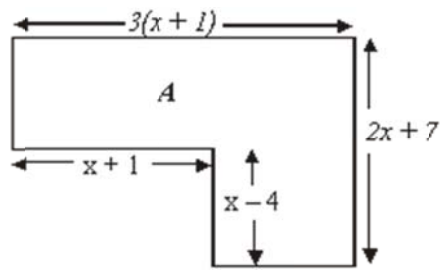
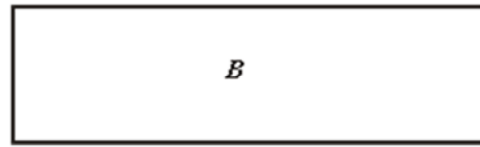


Diagram **NOT** accurately drawn



The diagram shows two shapes.
In shape *A*, all of the angles are right angles.
Shape *B* is a rectangle.
All the measurements are in centimetres.

The area of shape *A* is equal to the area of shape *B*.

Find an expression, in terms of x , for the length and an expression, in terms of x , for the width of shape *B*.

.....
(Total 6 marks)

Q12.

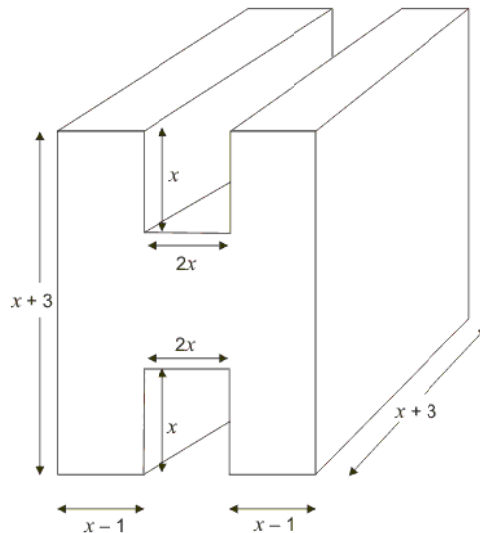


Diagram **NOT** accurately drawn

The diagram shows a prism. All measurements are in cm. All corners are right angles. The volume of the prism is $V \text{ cm}^3$.

Find a formula for V .

$V = \dots\dots\dots$

(Total 4 marks)

Q13. A water trough is in the shape of a prism.

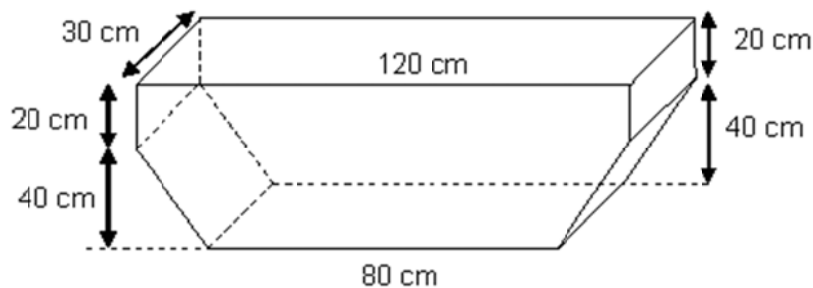


Diagram **NOT** accurately drawn

Hamish fills the trough completely.

Water leaks from the bottom of the trough at a constant rate.
2 hours later, the level of the water has fallen by 20 cm.

Water continues to leak from the trough at the same rate.

How many more minutes will it take for the trough to empty completely?

..... minutes

(Total 6 marks)

Q14. The diagram shows a pentagon.

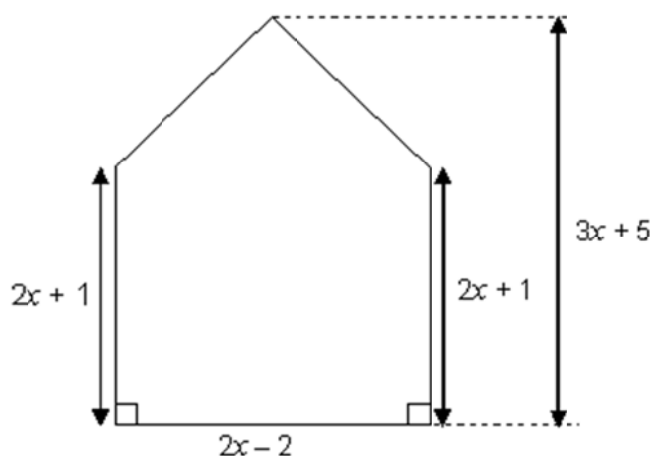


Diagram **NOT** accurately drawn

All measurements are in centimetres.

Show that the area of this pentagon can be written as $5x^2 + x - 6$

(Total 4 marks)

Q15.

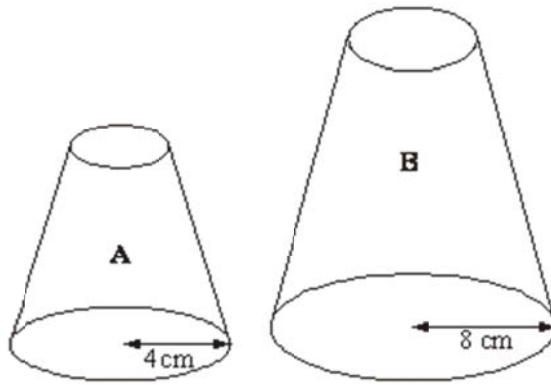


Diagram **NOT** accurately drawn

Two solid shapes, **A** and **B**, are mathematically similar.
The base of shape **A** is a circle with radius 4 cm.
The base of shape **B** is a circle with radius 8 cm.

The surface area of shape **A** is 80 cm².

(a) Work out the surface area of shape **B**.

..... cm²

(2)

The volume of shape **B** is 600 cm³.

(b) Work out the volume of shape **A**.

..... cm³

(2)

(Total 4 marks)

Q16.

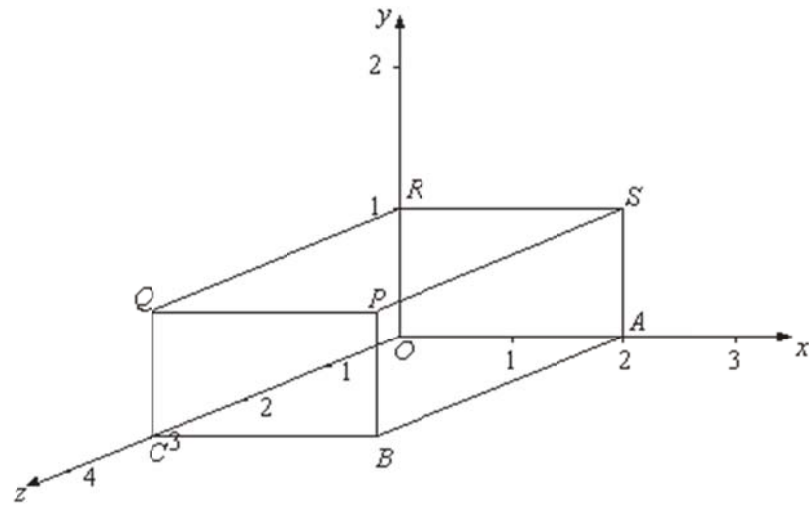


Diagram **NOT** accurately drawn

A cuboid is shown on a 3-dimensional grid.

- (a) Write down the letter of the point with coordinates (2, 1, 0).

.....

(1)

- (b) Write down the coordinates of the point *P*.

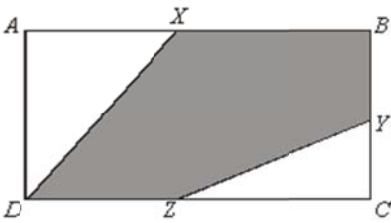
(..... , ,)

(1)

(Total 2 marks)

Q17.

Diagram **NOT** accurately drawn



ABCD is a rectangle.
X is the midpoint of *AB*. *Y* is the midpoint of *BC*. *Z* is the midpoint of *CD*.

What fraction of the total area of *ABCD* is shaded?

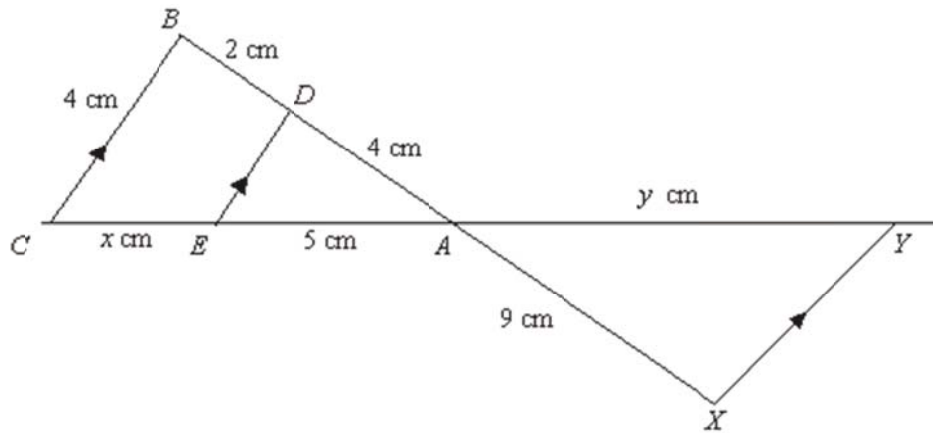
Show clearly how you get your answer.

.....

(Total 4 marks)

Q18.

Diagram **NOT**
accurately drawn



$CEAY$ and $BDAX$ are straight lines.

XY , ED and CB are parallel.

$AE = 5$ cm.

$AX = 9$ cm.

$AD = 4$ cm.

$BC = 4$ cm.

$BD = 2$ cm.

$CE = x$ cm.

$XY = y$ cm.

Find the value of x and the value of y .

$x = \dots\dots\dots$

$y = \dots\dots\dots$

(Total 4 marks)

Q19.

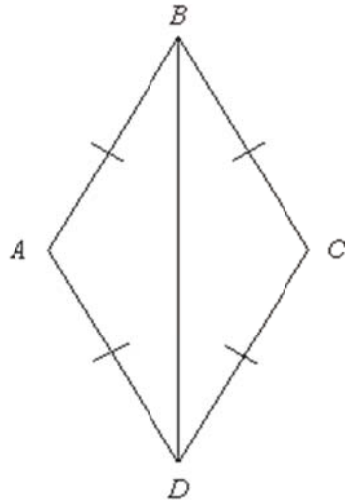


Diagram **NOT** accurately drawn

In the diagram, $AB = BC = CD = DA$.

Prove that triangle ADB is congruent to triangle CDB .

(Total 3 marks)

Q20.

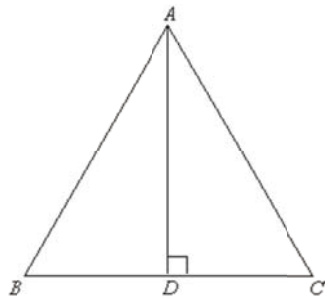


Diagram **NOT** accurately drawn

ABC is an equilateral triangle. D lies on BC . AD is perpendicular to BC .

(a) Prove that triangle ADC is congruent to triangle ADB .

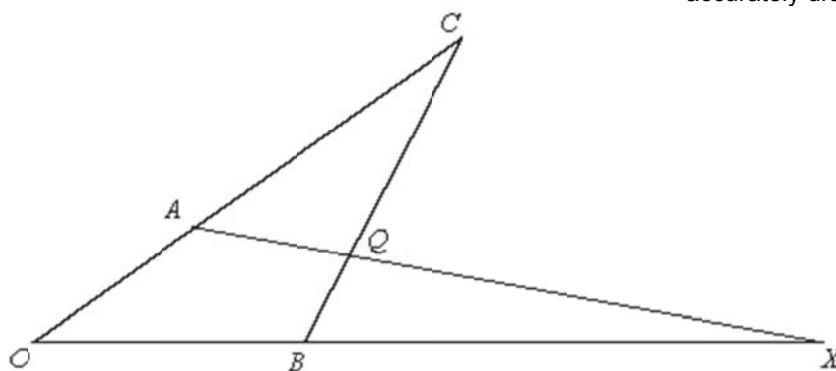
(3)

(b) Hence, prove that $BD = \frac{1}{2} AB$.

(2)
(Total 5 marks)

Q21.

Diagram **NOT**
accurately drawn



In the diagram,

$$\overrightarrow{OA} = 4\mathbf{a} \text{ and } \overrightarrow{OB} = 4\mathbf{b}$$

OAC , OBX and BQC are all straight lines

$$AC = 2OA \text{ and } BQ:QC = 1:3$$

(a) Find, in terms of \mathbf{a} and \mathbf{b} , the vectors which represent

(i) \overrightarrow{BC}

.....

(ii) \overrightarrow{AQ}

.....

(4)

Given that $\overrightarrow{BX} = 8\mathbf{b}$

(b) Show that AQX is a straight line.

(3)

(Total 7 marks)

M1.

	Working	Answer	Mark	Additional Guidance
(a)		(5, 2, 0)	1	B1 for (5, 2, 0) cao
(b)	$\left(\frac{0+5}{2}, \frac{2+0}{2}, \frac{3+3}{2}\right)$	$\left(\frac{5}{2}, 1, 3\right)$	3	B1 for (0, 2, 3) or for (5, 0, 3) or for (0, 0, 3) seen or implied $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$ M1 for $\left(\frac{5}{2}, 1, 3\right)$ oe A1 for $\left(\frac{5}{2}, 1, 3\right)$ oe B1 SC for (x, y, 3) <i>Alternative mark scheme</i> B1 for each coordinate correct.
Total for Question: 4 marks				

M2.

Working	Answer	Mark	Additional Guidance
Triangular face: $\frac{1}{2} \times 5 \times 12 = 30$ Rectangular faces: (13 × 10), (12 × 10), (5 × 10) Area: 30 + 30 + 130 + 120 + 50 =	360 cm ²	4	M1 for $\frac{1}{2} \times 5 \times 12 (= 30)$ oe M1 for 2 + of (13 × 10) and (12 × 10) and (5 × 10) oe A1 cao NB: No marks awarded for calculating volume B1 (indep) units stated (cm ²)
Total for Question: 4 marks			

M3.

	Working	Answer	Mark	Additional Guidance
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(i)	$180^\circ - 160^\circ$	20	1	B1 cao
(ii)	Exterior angles sum to 360° So $360 \div '20' =$	18	2	M1 for $360 \div "20"$ A1 cao
Total for Question: 3 marks				

M4.

Working	Answer	Mark	Additional Guidance
$180 - 140 (= 40)$ $360 \div "40"$	9	3	M1 for $180 - 140 (= 40)$ M1 (dep) for $360 \div "40"$ A1 cao
Total for Question: 3 marks			

M5.

Working	Answer	Mark	Additional Guidance
$\frac{1}{2} \times 6(10 + 8) - \frac{1}{2} \times 3(7 + 5)$ $= 54 - 18$	36	3	M1 for $\frac{1}{2} \times 6(10 + 8)$ or $\frac{1}{2} \times 3(7 + 5)$ oe M1 (dep) for $\frac{1}{2} \times 6(10 + 8) - \frac{1}{2} \times 3(7 + 5)$ oe A1 cao
Total for Question: 3 marks			

M6.

Working	Answer	Mark	Additional Guidance
8 km per 30 seconds $= 16$ km per minute $= 16 \times 60 = 960$ km per hour $960 \text{ km/hr} \times 5 \div 8 = 600$ miles per hour	600	3	M1 convert to km/h by $\times 2 \times 60$ or 960 seen or use of speed = distance \div time M1 convert distance to miles by $\times 5 \div 8$ oe or sight of 5 miles A1 cao

M7.

Working	Answer	Mark	Additional Guidance
$\pi x^2(2x) = \frac{1}{3} \pi(x)^2 h$	$6x$	3	<p>M1 for a correct volume formula in terms of x, e.g. $\pi x^2(2x)$ or $\frac{1}{3} \pi x^2 h$</p> <p>A1 for $\pi(2x) = \frac{1}{3} \pi h$ or $3\pi x^2(2x) = \pi x^2 h$ or $x^2(2x) = \frac{1}{3} x^2 h$ (or better)</p> <p>A1 for $6x$ cao</p>
Total for Question: 3 marks			

M8.

	Working	Answer	Mark	Additional Guidance
(a)(i)		$\frac{1}{2} \mathbf{a}$	2	B1 for $\frac{1}{2} \mathbf{a}$ oe
(ii)		$\frac{1}{2} \mathbf{a} - \frac{1}{2} \mathbf{c}$		B1 for $\frac{1}{2} \mathbf{a} - \frac{1}{2} \mathbf{c}$ oe
(b)	$\vec{CA} = \mathbf{a} - \mathbf{c}$ $\vec{MN} = \frac{1}{2} (\mathbf{a} - \mathbf{c})$	$\vec{MN} = \frac{1}{2} \vec{CA}$	2	<p>B1 for ($\vec{CA} = \mathbf{a} - \mathbf{c}$ or $\vec{CB} + \vec{BA}$) oe</p> <p>B1 (dep) for correct proof, e.g. '$\vec{CA} = 2\vec{MN}$', or '\vec{CA} is a multiple of \vec{MN}', (NB: condone absence/misuse of vector notation)</p>
				Total for Question: 4 marks

M9.

Working	Answer	Mark	Additional Guidance
Angle PQR = angle QRS = $\frac{(10-2) \times 180}{100} 144^\circ$ (interior angle of an nsided polygon) Angle QPR = angle QRP = $\frac{180 - 144}{2}$ = 18° (base angles of isos triangle) Angle PRS = 144 – 18 = 126° x = 180 – 126 = 54° (angles on a straight line)	54°	5	$\frac{(10-2) \times 180}{100} 144^\circ$ M1 for oe A1 for interior angle = 144 $\frac{180 - 144}{2}$ M1 for or 18° seen M1 (dep) for “180 – (‘144’ – ‘18’)” A1 cao
Total for Question: 5 marks			

M10.

	Working	Answer	Mark	Additional Guidance
QWC (i, ii, iii)	PS = PT and PQ = PR (equal tgts from a point) Let angle SPT = x Angle PST = angle PTS = $\frac{180 - x}{2}$ (base angles of isos triangle) Angle QPR = x (vertically opposite angles) Angle PQR = angle PRQ = $\frac{180 - x}{2}$ (base angles of isos triangle) Therefore angle PQR = angle PTS which are alternate angles. Hence QR is parallel to ST	Proof	5	B1 for PS = PT or PQ = PR B1 for equal tangents from a point B1 for angle PST = angle PTS = $\frac{180 - x}{2}$ or angle PQR = angle PRQ = $\frac{180 - x}{2}$ C1 for base angles of isos triangle are equal or vertically opposite angles are equal QWC: Working should be clearly laid out in a logical sequence, with calculations attributable C1 for alternate angles implying parallel QWC: Any technical language should be correct
Total for Question: 5 marks				

M11.

Working	Answer	Mark	Additional Guidance
$A = 3(x+1)(2x+7) - (x-4)(x+1)$ $= 3(2x^2 + 9x + 7) - (x^2 - 3x - 4)$ $= 5x^2 + 30x + 25$ Factorising gives $5(x+1)(x+5)$ OR Splitting shape A into rectangles, area to be added: e.g. $3(x+1)(x+11) + (x-4)(2x+2)$ $= 3(x^2 + 12x + 11) + (2x^2 - 6x - 8)$ $= 5x^2 + 30x + 25$ Factorising gives $5(x+1)(x+5)$	$5x+5$ by $x+5$ or $5x+25$ by $x+1$	6	M1 for attempting to subtract the area of small rectangle from area of large rectangle in A M1 for $3(x+1)(2x+7) - (x-4)(x+1)$ A1 for $3(2x^2 + 9x + 7)$ and $(x^2 - 3x - 4)$ A1 for $5x^2 + 30x + 25$ M1 for attempting to factorise " $5x^2 + 30x + 25$ " to get dimensions of B A1 for $5x+5$ by $x+5$ or $5x+25$ by $x+1$ OR M1 for attempting to add the area of two (or more) rectangles that make up the shape A M1 for $3(x+1)(x+11) + (x-4)(2x+2)$ oe equivalent A1 for $3(x^2 + 12x + 11)$ and $(2x^2 - 6x - 8)$ A1 for $5x^2 + 30x + 25$ M1 for attempting to factorise " $5x^2 + 30x + 25$ " to get dimensions of B A1 for $5x+5$ by $x+5$ or $5x+25$ by $x+1$
Total for Question: 6 marks			

M12.

Working	Answer	Mark	Additional Guidance
Unknown length = $x + 3 - x - x = 3 - x$ Cross-sectional area $= (x+3)(x-1) + (x+3)(x-1) + (3-x)(2x)$ $= x^2 + 2x - 3 + x^2 + 2x - 3 + 6x - 2x^2$ $= 4x - 6 + 6x$ $= 10x - 6$ Volume $= (10x - 6)(x + 3)$ $= 10x^2 + 24x - 18$ OR Unknown length = $x + 3 - x - x = 3 - x$	$10x^2 + 24x - 18$	4	B1 for $x + 3 - x - x$ oe or $3 - x$ seen or $x - 1 + 2x + x - 1$ oe or $4x - 2$ seen M1 for correct expression for 1 area from cross-section or for 1 volume of cuboid(s) (brackets not needed) M1 for correct method for total cross-sectional area OR at least 2 volumes added OR volume of surrounding cuboid – at least 1 vol (brackets needed)

Volume $= (x + 3)(x + 3)(x - 1) +$ $(x + 3)(x + 3)(x - 1) +$ $(2x)(3 - x)(x + 3)$ $= (10x - 6)(x + 3)$ $= 10x^2 + 24x - 18$ OR Unknown length $= (2x - 2) + 2x =$ $4x - 2$ Surrounding area $= (4x - 2)(x + 3) = 4x^2 + 10x - 6$ So $A = 4x^2 + 10x - 6 - 4x^2 =$ $10x - 6$ So $V = (10x - 6)(x + 3) =$ $10x^2 + 24x - 18$ OR Unknown length $= (2x - 2) + 2x =$ $4x - 2$ Surrounding volume $= (4x - 2)(x + 3)(x + 3)$ $V = (4x - 2)(x + 3)(x + 3) -$ $2x(2x)(x + 3)$			A1 for $10x^2 + 24x - 18$ oe
Total for Question: 4 marks			

M13.

Working	Answer	Mark	Additional Guidance
45	200 minutes	6	M1 for $120 \times 20 \times 30 (= 7200)$ M1 for $"72000" \div 120$ A1 for $600 \text{ cm}^3 \text{ min oe}$ M1 for $\frac{1}{2} \times (120 + 80) \times 40 \times 30 (= 120000)$ M1 for $"120000" \div "600"$ A1 for 200 minutes or 3 hours 20 mins oe SC B1 for 4 hours
Total for Question: 6 marks			

M14.

Working	Answer	Mark	Additional Guidance
$(2x - 2)(2x + 1)$ $+ \frac{1}{2} (2x - 2)((3x + 5) - (2x + 1))$ $4x^2 - 2x - 2 + x^2 + 4x - x - 4$ $= 5x^2 + x - 6$ Or $(2x - 2)(3x + 5)$ $- \frac{1}{2} (2x - 2)((3x + 5) - (2x + 1))$ $= 6x^2 - 6x + 10x - 10$ $- x^2 - 4x + x + 4$ $= 5x^2 + x - 6$	Show	4	<p>M1 for correct expression for a single rectangle area $(2x - 2)(2x + 1)$ or $(2x - 2)(3x + 5)$</p> <p>M1 for correct expression for triangle area $\frac{1}{2} (2x - 2)((3x + 5) - (2x + 1))$</p> <p>M1 for all 4 terms correct with or without signs or 3 out of no more than four terms correct with signs in expansion of any two linear expressions.</p> <p>C1 for $5x^2 + x - 6$ and all steps clearly shown in a logical progression</p> <p>QWC: All steps need to be clearly laid out showing a logical progression</p>
Total for Question: 4 marks			

M15.

	Working	Answer	Mark	Additional Guidance
(a)	$\left(\frac{8}{4}\right)^2 \times 80$	320	2	<p>M1 for $\left(\frac{8}{4}\right)^2$ or $\left(\frac{4}{8}\right)^2$ oe or $8^2:4^2$ or $4^2:8^2$ or $1:4$ or $4:1$</p> <p>A1 for 320 cao</p>
(b)	$\left(\frac{4}{8}\right)^3 \times 600$	75	2	<p>M1 for $600 \times \left(\frac{4}{8}\right)^3$ or $600 \times \left(\frac{8}{4}\right)^3$ oe</p> <p>A1 for 75 cao</p>

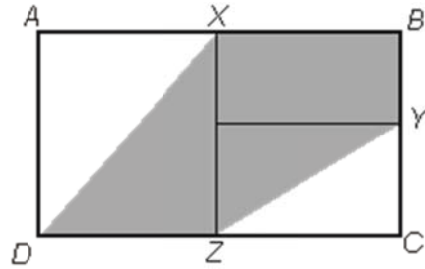
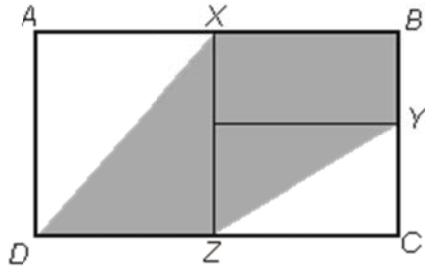
M16.

	Answer	Mark	Additional Guidance
(a)	S	1	B1 for S cao
(b)	(2, 1, 3)	1	B1 for (2, 1, 3) cao
Total for Question: 2 marks			

M17.

Working	Answer	Mark	Additional Guidance
<p>Let $AB = x$, $AD = y$</p> <p>Area of rectangle = xy</p> <p>Area $AXD = \frac{xy}{4}$</p> <p>Area $CYZ = \frac{xy}{8}$</p> <p>Shaded area = $\frac{5xy}{8}$</p>	$\frac{5}{8}$	4	<p>M1 a full method to find the unshaded area and subtracting from 1</p> <p>B1 area of $AXD = \text{area of } ABCD \div 4$</p> <p>B1 area of $CYZ = \text{area of } ABCD \div 8$</p> <p>A1 cao</p> <p>OR</p> <p>Diagram</p> <p>M1 for dividing left into 2 congruent triangles for dividing right into 4 congruent triangles</p> <p>B1 left = $2A$ and $2A$ or shaded = $\frac{1}{2}$ or $\frac{1}{2} = \frac{1}{4} = \frac{2}{8}$</p> <p>B1 right = $2A$ and A and A or shaded = $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$</p> <p>A1 cao</p> <p>Substitution</p> <p>M1 for deciding upon suitable side lengths for AD and AB and calculating dimensions of internal shapes</p> <p>B1 for area of DZX</p> <p>B1 for area of $ZXBY$</p> <p>A1 cao</p>

			OR M1 for deciding upon suitable side lengths for AD and AB and calculating dimensions of internal shapes B1 for area ADX B1 for area ZCY A1 cao
Total for Question: 4 marks			



M18.

Working	Answer	Mark	Additional Guidance
$\frac{x}{5} = \frac{2}{4}$ $\frac{y}{x+5} = \frac{9}{6}$ or $\frac{y}{9} = \frac{x+5}{6}$	$x = 2.05$ $y = 11.25$	4	M1 a correct expression for x involving ratios of sides, e.g. $\frac{x}{5} = \frac{2}{4}$ oe A1 cao M1 $\frac{y}{x+5} = \frac{9}{6}$ or $\frac{y}{9} = \frac{x+5}{6}$ oe OR $\frac{y}{5} = \frac{9}{4}$ A1 cao
Total for Question: 4 marks			

M19.

Working	Mark	Additional Guidance
$AD = CD$ equal sides $AB = CB$ equal sides BD is common ADB is congruent to CDB (SSS)	3	B2 for two of $AD = CD$, $AB = CB$, BD is common OR for BD common and all other sides equal in length (it must be clear that the 'other sides' relate to the two triangles) (B1 for one of these. Note: All sides are of the same length alone is ambiguous and gains B0) B1 for proof of congruence (SSS or SAS or ASA) dependent upon THREE identities (with reasons)
Total for Question: 3 marks		

M20.

	Working	Answer	Mark	Additional Guidance
(a)	$AB = AC$ (equilateral triangle) AD is common $ADC = ADB$ ($= 90^\circ$ given) $\triangle ADC \equiv \triangle ADB$ (RHS) OR $DAC = DAB$ (since $ACD = ABD$ and $ADC = ADB$) $AB = AC$ (equilateral triangle) AD is common $\triangle ADC \equiv \triangle ADB$ (SAS) OR $DAC = DAB$ (since $ACD = ABD$ and $ADC = ADB$) AD is common $ACD = ABD$ (equilateral triangle) $\triangle ADC \equiv \triangle ADB$ (AAS)	Proof	3	M1 for any three correct statements (which do not have to be justified) that together lead to a congruence proof (ignore irrelevant statements) A1 for a full justification of these statements A1 for RHS, SAS, AAS, ASA or SSS as appropriate NB The two A marks are independent
(b)	$BD = DC$ (congruent Δ s) $BC = AB$ (equilateral Δ s) $\text{Hence } BD = \frac{1}{2} AB$	Proof	2	B1 for $BD = DC$ and $BC = AB$ B1 for justification of these statements and completion of proof
Total for Question: 5 marks				

M21.

	Working	Answer	Mark	Additional Guidance
(a) (i)	$\overline{BC} = \overline{CO} + \overline{OB}$ $\overline{AQ} = \overline{AO} + \overline{OB} + \overline{BQ}$	$12a + 4b$ $3b - a$	4	M1 $\overline{BC} = \overline{CO} + \overline{OB}$ A1 cao
(ii)	$= -4a + 4b = \frac{1}{4}(12a + 4b)$			M1 $-4a + 4b = \frac{1}{4}$ '(12a + 4b)' A1 cao
(b)	$\overline{OX} = 12b$, $\overline{AX} = -4a + 12b$ $= 4(-a + 3b)$	Correct reason, with correct working	3	B1 $\overline{OX} = 12b$ B1 $\overline{AX} = -4a + 12b$ C1 convincing explanation
Total for Question: 7 marks				